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COMMENT

Position-space renormalisation group approach for bond lattice animals

N-C Chao†, C-N Chen, K Y Lin and Felix Lee

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 30043, Republic of China

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Abstract. We perform two position-space renormalisation group calculations based on a 'centre rule' weight function for bond lattice animals on the square lattice. The critical exponent ν is determined within the range 0.6430-0.6528.

Position-space renormalisation group (PSRG) techniques have been successfully applied to various lattice statistical problems that are related to polymeric systems (see, for example, Stanley *et al* 1982). In particular, the branched polymers as modelled by lattice animals have been treated by Family (1980, 1983) for both the site and the bond problems. However, despite the success of the PSRG treatment for site animals in comparison with other methods, the treatment for bond animals is still unsatisfactory.

For the bond problem, Family has adapted the 'corner rule' weight function (de Queiroz and Chaves 1980), in which the origin is chosen at a corner of the cells similar to those shown in figure 1, and obtained $\nu = 0.627$ based on a cell-to-cell renormalisation. In this comment, we present two calculations both using a 'centre rule' weight function on the square lattice (Chao 1985, Lin *et al* 1990). In the first calculation, we use the family of cells illustrated in figure 1(a), which is exactly the same one used for the self-avoiding walk problem (Chao 1985). The generating functions for $b = 2, 3$ and 4, which are the sum of statistical weights for all spanning animals along the top-bottom direction of the cells, can be evaluated, respectively, as

$$G_2(K) = K^5 + 4K^4 + 4K^3 + K^2 \quad (1)$$

$$G_3(K) = K^{13} + 13K^{12} + 72K^{11} + 217K^{10} + 397K^9 + 456K^8 + 334K^7 + 162K^6 + 58K^5 + 12K^4 + K^3 \quad (2)$$

$$G_4(K) = K^{25} + 25K^{24} + 296K^{23} + 2197K^{22} + 11\,399K^{21} + 43\,628K^{20} + 126\,764K^{19} + 284\,127K^{18} + 496\,240K^{17} + 679\,947K^{16} + 734\,190K^{15} + 627\,534K^{14} + 428\,093K^{13} + 236\,881K^{12} + 108\,846K^{11} + 42\,010K^{10} + 13\,577K^9 + 3650K^8 + 804K^7 + 138K^6 + 16K^5 + K^4 \quad (3)$$

† On leave from Universidade Federal do Rio Grande do Norte, Departamento de Física, Natal, Rio Grande do Norte, Brazil.

where K is the fugacity per monomer. The remormalisation of such cells to a bond with fugacity K' :

$$G_b(K) = K' \tag{4}$$

can be solved to obtain fixed points as $K = K' \equiv K_b^*$. The results are

$$K_b^* = \begin{cases} 0.3451 & \text{for } b = 2 \\ 0.2675 & \text{for } b = 3 \\ 0.2440 & \text{for } b = 4. \end{cases} \tag{5}$$

Linearising G_b at K_b^* , we can calculate ν_b from the formula

$$\nu_b = \frac{\ln b}{\ln \lambda} \tag{6}$$

with the eigenvalue $\lambda = (dG_b(K)/dK)|_{K^*}$ (see Stanley *et al* 1982). We obtain

$$\nu_b = \begin{cases} 0.6624 & \text{for } b = 2 \\ 0.6604 & \text{for } b = 3 \\ 0.6528 & \text{for } b = 4. \end{cases} \tag{7}$$

It is the characteristic of the ‘centre rule’ weight function that the family of cells with b odd and even should be grouped separately due to difference in the location of the origin. The results for b even indicate that ν_b seems to be a decreasing function of b . Thus ν_4 may be considered as the upper bound in our calculation.

In the second calculation, we use the family of cells shown in figure 1(b), which differs from figure 1(a) in that a line of bonds is added to the bottom edge of cells. Although such cells may not be suitable for the self-avoiding walk problems due to the formation of ramified configurations at the ‘interfacing’ between cells, such an objection is irrelevant for lattice animals. The generating functions for $b = 2$ and 3 become

$$G_2(K) = K^6 + 6K^5 + 10K^4 + 5K^3 + K^2 \tag{8}$$

$$G_3(K) = K^{15} + 15K^{14} + 103K^{13} + 417K^{12} + 1072K^{11} + 1768K^{10} + 1912K^9 + 1384K^8 + 737K^7 + 290K^6 + 83K^5 + 14K^4 + K^3. \tag{9}$$

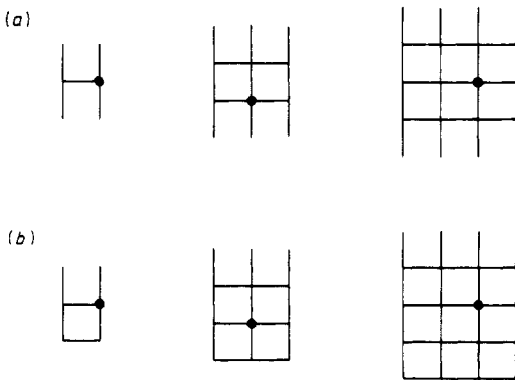


Figure 1. The family of cells adapted for the first (a) and the second (b) calculations. Cells with size $b = 2, 3$ and 4 are illustrated.

The expression for $G_4(K)$ is too tedious to be presented here. From these new generating functions together with equations (4) and (6), we obtain another set of results:

$$K_b^{**} = \begin{cases} 0.2902 & \text{for } b = 2 \\ 0.2406 & \text{for } b = 3 \\ 0.2279 & \text{for } b = 4 \end{cases} \quad (10)$$

and

$$\nu'_b = \begin{cases} 0.6224 & \text{for } b = 2 \\ 0.6432 & \text{for } b = 3 \\ 0.6430 & \text{for } b = 4. \end{cases} \quad (11)$$

The results (11) indicate that ν'_b for b even is now an increasing function of b and ν'_4 may be considered as the lower bound. Thus, we conclude that the value for ν may be restricted within the range

$$0.6430 < \nu < 0.6528. \quad (12)$$

This result is consistent with $\nu = 0.649$ of Family and others (see Family 1983).

From the cell-to-cell renormalisation scheme, we obtain $\nu_{4/2} = 0.6272$ and 0.6465 for the first and the second calculation, respectively. However, since we have only one value for each case, such results are less conclusive.

Finally, we mention that although the cells we used differ by a line of bonds at the bottom edge, the effect of such difference at the surface should decrease as the cell size increases.

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